Joe Bustillos
ED 502 Teaching Reading
07-21-93
Journal Articles: Teaching for Understanding: Three Key Attributes, by Richard S. Prawat.

And the epistemological wheels keep on turning. This time Prof. Prawat focused in on three attributes that need to be addressed if teachers want to teach for understanding. The three attributes are Focus/Coherence, Negotiation, and Analysis/Diagnosis.

I don't know. I've seen a huge push for "hands-on" math & science

What was most interesting about this article was that Prawat focused in on math and science to critique current teaching methods, two areas that are generally the bread and butter of traditionalist teaching. It's a stereotype, but "student-centered" constructivist curriculum has generally focused in on the liberal arts/creativity-based subjects---literature, reading, and art. And the temptation has been to fall back on the old methods when dealing with content-heavy subjects such as math and science. But this, according to Prawat, is just when teacher's need to be more student centered.

Oh I absolutely agree w/ this

Do you think this has anything to do w/ testing?

According to Prawat's analysis the weak link is the teacher's comfort-level with the subject being presented. If the teacher is not entirely sure of his/her understanding of the subject then there is going to be a tendency to be linear in ones explanation and unwilling or unable to address the student questions that are inevitable in teaching. Another liability according to Prawat is teaching for quantity instead of quality. This goes hand-in-hand with allowing for student "interruptions" in that greater long-term success is fostered by ensuring greater

depth of understanding rather than being so intent on "getting through the material." His point is well-made when he compares the Japanese method of spending a whole class period on two to four math problems to ensure understanding rather than the typical American technique of bulldozing through fifty problems that the students are or aren't getting (because we have to keep "on task"). In the long run content areas are not sacrificed because less time is spent going "back over what we 'learned' last Friday about multiplication with decimals," for example.

I hope this is changing, but it is very common. Kids do 100 problems mechanically without really understanding what they're doing

On other element of Prawat's article is that all student's do not learn math or science the same way. Therefore becoming a slave to artificial hierarchy of mathematic concepts ("first single digit adding, then single digit subtraction . . .) is ill-advised. It's a bit like teaching only 19 letters of the alphabet because the students "can't handle it." Placing the subject matter back in context tends to alleviate these memory/understanding problem. Hmmm, that sounds just like reading.

Isn't it interesting how you start to make these connections?

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TEACHING FOR UNDERSTANDING: THREE KEY ATTRIBUTES*

RICHARD S. PRAWAT

Michigan State University, U.S.A.

Abstract — Assumptions about teaching and learning play a vital role in any attempts to improve young students' mathematics and science performance. Despite the recent efforts of many educators and psychologists, "absorptionist" views of learning and "transmission" views of teaching continue to prevail. This paper draws on current research to argue for a more conceptually oriented approach to teaching. Such an approach, which highlights the role of three key attributes — focus, negotiation, and analysis — is more consistent with the emerging view that stresses the importance of developing networks of knowledge in learning mathematics and science.

Concern in the U.S.A. about the adequacy of student learning in important subjects like mathematics and science has been fueled recently by well-publicized findings from a number of national surveys. A summary of four of these surveys (Dossey, Mullis, Lindquist, & Voss, 1988) shows that, although students in the United Sates have mastered simple arithmetic facts, only a small percentage are capable of complex, multi-step reasoning in mathematics. Several international studies point to the same problem: U.S. students lag far behind students in other countries in mathematics and science, particularly on items that measure complex skills and understanding (Husen, 1983) — despite an increase in the amount of homework and testing in these subjects in recent years (Dossey, Mullis, Lindquist, & Chambers, 1988). The comparatively poor performance of U.S. students has both personal and national implications.

In the next 20 years the fastest growing areas of employment will require employees to have much higher mathematics, language, and reasoning capabilities than currently is the norm (Hudson Institute, 1987). Because the U.S. economy is becoming more and more global in nature, American workers will be competing with workers in other countries for these jobs (National Commission on Excellence in Education, 1983). Unless we in the U.S.A. dramatically improve the quality of our education, our workers may not fare well in this competition. Evidence for this contention abounds: For example, the *Wall Street Journal* reported several months back that a Japanese semiconductor company, opening a plant in the southeastern United States, was forced to use graduate students to do a job that was performed by high-school graduates in Japan (Dossey, Mullis, Lindquist, & Chambers, 1988).

Arguably, much of the blame for our students' poor performance in mathematics and science can be attributed to the way we teach these subjects. For the most part, this instruction emphasizes factual and procedural knowledge at the expense of conceptual understanding (Stake & Easley, 1978). The focus of factual

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and procedural knowledge is not unique to mathematics and science, of course. It is consistent with our whole approach to education. As David Cohen (1988) explains:

> Contemporary instructional practices embody an ancient instructional inheritance. In this inheritance, teachers are active; they are tellers of truth who inculcate knowledge in students. Learners are relatively passive; students are accumulators of material, who listen, read, and perform prescribed exercises. And knowledge is objective and stable. It consists of facts, laws, and procedures that are true, independent of those who learn, and entirely authoritative. (p. 15)

Few would deny that this characterization applies to mathematics and science teaching in the U.S.A. In mathematics, the content is primarily low level, particularly in elementary school (Peterson, 1988), and seatwork predominates — accounting for approximately 75% of the instructional time typically devoted to mathematics (Denham & Lieberman, 1980). Nor is the situation much different in science. *When* science is taught at the elementary school level, a recitation format is used, with the sequencing of topics often being dictated entirely by the textbook (Stake & Easley, 1978; Weiss, 1987). Using textbooks creates problems: They are not good vehicles for fostering conceptual level understanding in students because they attempt to cover too much material, treating individual topics in a superficial fashion (Newman, 1988).

The use of better curriculum materials would alleviate some of the problem in mathematics and science (California State Department of Education, 1985); this would not be a panacea, however. Educators are more sophisticated now than they were 20 years ago about the role that such material plays in instruction. Research demonstrates that even the best curriculum material is modified to fit practitioners' views of teaching and learning (Carlsen, in press; Hashweh, 1985; Smith & Neale, in press). It is these views of teaching and learning, then, that need to be changed if teaching practice is to change. Unfortunately, teacher educators have not demonstrated much capacity to promote this sort of change (Tabachnick & Zeichner, 1984). Fundamental issues of schooling, learning, and knowing are seldom adequately addressed in preservice and inservice teacher education programs (see Zeichner & Liston, 1987, for an exception).

The purpose of this paper is to deal with these central issues. The assumptions we make about teaching and learning will play a decisive role in any efforts to improve students' mathematics and science performance in the U.S.A. and elsewhere. For this reason, it makes sense to start with the most basic of questions: What does it mean to teach for conceptual level understanding? How does it differ from what we currently do? With the hope of provoking further thought and discussion about this issue, an alternative — and more conceptually oriented — view of teaching will be presented. This view is derived from current research, and highlights the role of three key attributes or features that distinguish teaching for understanding from more traditional approaches to instruction. Before elaborating on this view, however, it may be helpful to consider some of the problems associated with a more traditional approach to teaching. These are best illustrated using a concrete example.

Miss Jones works with fifth graders in a predominantly middle class school located in the midwest. She is a good teacher by most criteria; for example, her class is well managed and she has good rapport with her students, as illustrated in the following field-notes:

> The focus of this lesson is on multiplying decimals. Prior to assigning students a series of seatwork problems, Miss Jones is working through two problems on the board. One example involves money — \$32.45 × 0.5. The teacher repeats the algorithm that students are to use in doing these problems: First, multiply as you would with whole numbers. Second, count the number of places to the right of the decimal point in the top number, then count the number of places to the right of the decimal point in the bottom number. Add these together, and place the decimal point so that the product contains this number of places. Most students seem to understand the procedure and are anxious to get started, when one little boy raises his hand. He has a perplexed look on his face. Suddenly he blurts out, "This doesn't make sense. You started with 32 dollars and 45 cents and ended up with 16 dollars and whatever cents. You multiplied by that number [0.5], how did you get *less*?" Two other children agree that it does not make sense. At first the teacher thinks she has made a computational mistake, so she works the problem again. It soon becomes obvious that this is not the problem. It is more basic than that: The little boy thinks that the product should be bigger, not smaller than the

number that is multiplied. The teacher is at a loss about how to handle the question. She repeats the algorithm, explaining to the class, "I'm just teaching the computational way. What I'm looking for now is for everyone to understand where to place the decimals." The little boy shakes his head, still confused. The teacher assigns the problem set.

After class, I had a chance to talk to Miss Jones about her teaching and she expressed what I think are some fairly typical views about mathematics. She said:

> It's important at the elementary level to stress the basics. I don't have time for all the fancy stuff — problem solving and estimating. Kids have to first learn how to do computations. Even then, you have to keep pounding it into their heads. Later on, at the junior high level, it'll start to make sense to them.

I specifically asked Miss Jones about her decision to move on with the lesson even though some of the students seemed confused, and she talked about how much material they had to cover in mathematics. She indicated that her group was almost one chapter behind where it needed to be to finish the book by the end of the year.

This vignette illustrates several of the problems we face if we are going to promote mathematical understanding in students: One of the most obvious is the emphasis on breadth of coverage instead of depth. One can detect in Miss Jones' comments a certain amount of conflict over this issue, which is consistent with Newman's (1988) findings. He reports that teachers are under considerable external pressure to "cover" a prescribed amount of material. In addition to tests and district guidelines, textbooks are major culprits in this regard. More topics are included in textbooks than teachers can adequately treat. Critics of this material refer to this as the "mentioning" problem. As they point out, textbooks jump from fact to fact, topic to topic, without getting into sufficient depth in any one area to foster real understanding (Calfee, 1987).

The thing we now realize is that it takes *time* to foster understanding, partly because students develop powerful ideas of their own that frequently interfere with what we want them to learn. In an elementary science study, students were followed through a unit on photosynthesis. Despite 8 weeks of traditional instruction on this topic, only 7% of a large sample of fifth graders understood at the end of the unit that plants get their food by making it themselves. Most held on to their original beliefs: that plants, like people, ingest food from the outside (Roth & Anderson, in press). Learners are not blank slates. They construct their own understanding and this may or may not be consistent with what we are trying to get them to learn. The little boy in the example above who believes that multiplication means things get bigger is an example. Teachers must get in touch with this informal knowledge and figure out how to connect it with the new knowledge that we want students to acquire.

Another major obstacle to promoting understanding is the commonsense notion that learning progresses hierarchically. According to this view, the mastery of certain "prerequisite," lower order facts and skills is a necessary if not sufficient condition for the development of more complex understanding and application-oriented learning. The pyramid is an apt metaphor for this view of learning. Both the model and the view, however, are at variance with more recent research that stresses the importance of connections — of developing networks of knowledge (Prawat, in press). A spider web, or tinker toys with nodes and connectors, are metaphors that better capture the essence of this type of learning. Research demonstrates that we can help students make connection from the very beginning. This connecting can take many forms: Reconciling formal knowledge with the informal knowledge students develop on their own; linking key concepts and principles in mathematics and science to physical representations, models, metaphors, and analogies; demonstrating how separate concepts and rules — for example, the rules for computing the area of a triangle and of a rectangle — are interrelated. One mathematics educator calls this last type of connection "relational understanding" (Skemp, 1978) and argues that, although it is harder to learn, it is much easier to remember.

Similar to our use of the lower order, higher order distinction is our tendency to separate content and process. In many textbooks, "thinking skills" are dealt with in isolation from subject matter content. Resnick (1987a) addresses this isolation, arguing that so-called higher

order thinking skills must be embedded within school disciplines. "Paradoxically," she writes, "dropping the quest for general skills might, in the end, be the most powerful means of cultivating generally higher levels of cognitive functioning (p. 36). Recent research supports this notion: Experts are better problem solvers than novices *not* because they have mastered a set of general thinking skills but because they know more about certain things than novices (Chi, Feltovich, & Glaser, 1981). Experts make greater use of a few important ideas that can be used in a variety of contexts. In mathematics, an example might be the notion that only "like things" can be added together (Hiebert & Lefevre, 1986). This idea is useful in problems involving *both* the addition of fractions and the addition of decimals.

Our ability to draw on previous knowledge in new situations is also very much influenced by how it is organized. According to Polya (1973), this factor is even more important than the extent of one's knowledge. A major source of the difference between experts and novices is the way the former are able to organize their knowledge in a domain so that it can be used efficiently and effectively (Sternberg, 1981). Some ideas serve as better organizers than others. Identifying and figuring out how to focus our instruction on these "key ideas" is one of the major challenges confronting those who want to move toward a more conceptual approach to teaching (Prawat, in press).

The problems identified above can be attributed to certain widely held but questionable assumptions about the teaching/learning process. Not surprisingly, these assumptions are based on views of teaching and learning that tend to be mutually reinforcing: The widely accepted "absorptionist" view of learning (i.e., the belief that individuals learn by absorbing new information) and the equally popular "transmission" view of teaching are clearly related. Both prevail despite the recent efforts of many educators and psychologists. Paradoxically, in these views of teaching and learning, both teacher and student play relatively passive roles — the teacher as dispenser of information, the student as repeater (Lockhead, 1985). Doyle (1986), Cohen (1988), and others document the vested interests of both students and teachers in maintaining this status quo.

Although the transmission view of teaching is the most common, what Bereiter (1985) terms the "nonspecific" approach to teaching runs a close second. The focus in this type of teaching is on general instructional processes (e.g., inquiry) thought to exert an important influence on learning. The nonspecific approach has its adherents, particularly in science. It also has its critics. Anderson and Smith (1987), for example, have expressed concern about the excessive emphasis placed on process in so-called discovery approaches to science. They emphasize that process skills such as observing, measuring, and making inferences, which form the core of discovery-oriented curricula in science, were developed as a means to an end — the end being a better understanding of the how the world works. In many activity-based programs, Anderson and Smith argue, there is too little focus on conceptual understanding. Teachers who favor this approach apparently assume that children will assimilate science content directly from experience (Smith & Anderson, 1984). They often fail to provide the requisite amount of instructional support. As Smith and Neale (in press) explain, "Discussions are often deleted because teachers assume that children have correctly induced the truth as revealed in the empirical phenomena." As a result, students often rely on their own misconceptions to interpret activities and experiments.

Bereiter (1985) compares the nonspecific approach to teaching to an exercise and diet program in health. It deals with factors that are, in a sense, one step removed from the content learning process. He prefers an alternative that falls somewhere between the two extremes discussed above. In this middle-ground approach, instructional strategies function more like enzymes and hormones: They play a specific role in learning, even though their influence is indirect. The instructional interventions are indirect because the goal is to get students to construct their own knowledge. As Resnick (1988) puts it, the task is "to develop a psychology of instruction that places the learners' active mental construction at the heart of the instructional exchange" (p. 47). The teacher's task is to create conditions that allow students to construct knowledge that is both powerful and "correct" (i.e., consistent with disciplinary knowledge). How might one characterize such

interventions, which are more likely than traditional approaches to promote conceptual level understanding in students? Current research points to three attributes that appear to be of central importance in attempts to teach for understanding. The instruction should be focused and coherent, it should be negotiatory in its interactive style, and it should be strongly analytic or diagnostic on the teacher's part.

Focus and Coherence

Support for the importance of focus and coherence in teaching for understanding comes from several sources. First, there is the expert–novice research, which indicates that the expert's knowledge base is organized around a more central set of important ideas or understandings than the novice's. Specifically, experts are said to possess "multilevel" knowledge structures (Bereiter & Scardamalia, 1986); presumably, middle-range "key ideas" play a major role in this regard. An example in mathematics might be the notion of additive composition, the principle that all quantities are compositions of other quantities, or the principle of partition, which allows one to recompose problems into sets of more easily manipulated subproblems and then to cumulate the partial results (e.g., 65−23 converted to a more solvable [60−20]+[5−3]). By implication, expertise in students may be best fostered when school curricula carefully attend to a network of central ideas or understandings derived from the disciplines.

A second, more direct source of support for this notion is the growing body of research relating teachers' subject matter understanding to students' subject matter understanding. Until recently, this kind of research was virtually nonexistent (Shulman, 1986), partly because of the focus on generic teaching processes such as classroom management (Shulman, 1987), and partly because it did not seem profitable. As Ball (in press) points out, earlier research had failed to demonstrate any consistent relationship between student achievement and teacher knowledge in various subject matter domains; however, this research relied on indirect measures of teacher subject matter knowledge, such as the number of college-level courses taken in a particular domain.

Thanks in large part to the recent emphasis on conceptual understanding and higher order thinking in students, particularly in mathematics and science, the role of teacher content knowledge is being reexamined. This research is demonstrating that there is a clear relationship between what teachers know about content and the depth of understanding they are able to promote in students. This relationship is far from perfect; other variables influence the extent to which teachers utilize their content knowledge. As Ball (in press) argues with regard to mathematics, "A teacher who does understand the role of place value and the distributive property in multiplying large numbers will not necessarily draw upon this understanding in her teaching, for her ideas about learners or about learning may intervene." Teachers with the same level of conceptual understanding may teach differently depending upon their educational beliefs (i.e., their beliefs about teaching and learning). Nevertheless, a good grasp of what ideas are most central to the discipline and how they relate to one another bears a necessary if not sufficient relationship to conceptual level teaching.

Research conducted by Lee Shulman and his colleagues at Stanford University provides support for this notion. Steinberg, Haymore, and Marks (1985), for example, intensively followed four secondary mathematics teachers in their first year of teaching. Based on interview and observation data, they developed detailed case studies of these teachers that focused, in particular, on the relationship between content knowledge and teaching practice. The two teachers who had the surest grasp of mathematics — being able to identify central ideas and relate concepts — were also the most "conceptual" in their approach to teaching. They were more inclined to explain why certain mathematical procedures do or do not work, to stress central ideas, and to engage the students in more problem solving activity. This ability to focus on the big picture is characteristic of expert teachers in other subject matter domains as well (Gudmundsdottir & Shulman, 1987).

Closely related to content knowledge, and playing an equally important role in instruction, is a special type of knowledge that allows teachers to transform what they know into

something meaningful for students. Termed "pedagogical content knowledge" (Shulman, 1986), it includes vital things like understanding how particular ideas will be constructed by students — what sorts of preconceptions or misconceptions might have to be dealt with when introducing new material, which forms of representation are most useful for getting certain ideas across, how students will respond to certain activities and subject matter material. Especially relevant to the issue of focus and coherence, pedagogical content knowledge also includes knowledge about key ideas in a subject matter domain: Which are the most important ideas for students to understand, and how can they be best organized and sequenced to maximize student understanding?

Once termed the "missing program" in research on teaching (Shulman, 1986), work aimed at understanding the effects of subject-matter and pedagogical content knowledge on teaching has become popular of late (Carlsen, in press; Hashweh, 1985; Smith & Neale, in press). Leinhardt's research (1988) is but one example of recent case studies that document the importance of conceptual focus in teaching for understanding. She did a detailed analysis of one expert second-grade teacher's subtraction lessons. In this study, teacher subject-matter knowledge was examined in a novel way: Eight lesson videotapes were transcribed, and each conceptual statement made by the teacher was analysed using a mapping technique similar to Novak and Gowin's (1984). Interestingly, the same set of five key ideas was found in each of the eight lesson diagrams; an example is the notion that certain subtraction problems (called "foolers", because the top number in the "ones" place is smaller than the bottom) require different treatment. This teacher, who was successful in getting students to understand the mathematical basis for regrouping, had focused her instruction on a limited set of major ideas.

Support for the importance of focus and coherence also comes from cross-cultural studies of mathematics teaching and learning. Stigler and Perry (1988), for example, have compared the way mathematics is taught in Japanese, Taiwanese, and American classrooms. Although pointing out the preliminary nature of their findings, Stigler and Perry cite evidence supporting the contention that the mathematics curricula of these two Asian countries are more focused and coherent than the American. The Asian teachers appeared to provide students with more opportunities to make connections across elements of segments of mathematics lessons: "In Chinese classrooms, and in Japanese classrooms to an even greater extent, we see teachers providing explicit markers to aid children in inferring the coherence across different segments within a lesson, and across different lessons" (pp. 42–43). This attempt to provide coherence is not as evident in classes observed in the United States. Stigler and Perry speculate that it may be easier for such Asian teachers to make connections of this sort because their lessons tend to be much more focused than those observed in our own country. It is not uncommon, they report, for teachers in Japan and Taiwan to devote an entire 40-minute mathematics class to working two or three problems (i.e., discussing alternative solutions, etc.). These differences in coherence and focus may play an important role in accounting for the Asian students' mathematical superiority *vis-à-vis* the American.

One final bit of evidence for the importance of focus and coherence comes from an exciting new study by Newman and his colleagues (Newman, 1988). Their research examines factors that impede and facilitate higher order thinking in high school social studies classes. Of particular relevance here is the instrument they recently developed to measure the amount of "thoughtfulness" evident in classroom discourse. Two of the six scales on this instrument reflect focus and coherence concerns: one assesses the extent to which discourse is characterized by "sustained examination of a few topics rather than superficial coverage of many;" the second scale, titled "substantive coherence and continuity," gets at the extent to which ideas are pulled together or integrated during discourse.

Negotiation

Several cognitive psychologists, especially in mathematics, have used the term "negotiation" to describe the kind of interaction that occurs between teacher and student and between student and student in classrooms where teaching

for understanding is the norm (Cobb, Yackel, & Wood, 1988, in press; Schoenfeld, in press; Steffe, 1988). Use of this term highlights the social nature of the learning process, particularly if one focuses on *one* of at least two possible definitions. According to this first definition, negotiation is a process of reasoning together; when one negotiates, one confers with others in order to reach agreement on some important matter.

This definition fits well with the dialogic nature of conceptual learning; the importance of discourse processes in promoting conceptual level understanding and higher-order thinking is becoming increasingly apparent in the research literature (Brophy, in press). This definition misses the mark in another way, however. It suggests that knowledge can be created through consensus or a type of bargaining process in the classroom. This relates to an important epistemological problem: Is knowledge historical artifact or universal truth? The straw man position on either side of this issue has us, on the one hand, haggling over truth, and on the other, accepting the voice of authority. It is possible, however, to look at the problem from a different vantage point.

Cobb et al. (1988) suggest combining psychological and anthropological perspectives. Individuals construct their own reality, but this reality must be consistent with that shared by members of the disciplinary community if one is to participate in the discourse of that community. One goal of education is to enculturate students into the various disciplinary communities. Members of these communities share certain beliefs: for example beliefs about what constitutes a plausible argument in the context of a discipline like mathematics or science. These institutionalized beliefs constitute disciplinary knowledge. Because the teacher's task is enculturation — which involves not only intellectually challenging the child but also seeking to "shape" the child's knowledge in certain ways — a fair amount of tension is inherent in the teacher's role. Again, speaking of mathematics, Cobb et al. (1988) assert, "It is the tension between encouraging students to build on their informal ways of knowing and attempting to teach them the institutionally sanctioned formal knowledge of codified academic arithmetic that gives rise to the paradox of teaching" (p. 3).

Negotiation in the classroom, then, involves more than reaching agreement on important matters; as the above comments suggest, it also involves moving students in a certain direction (i.e., toward the view of reality shared by those in the diciplinary community). Defining negotiation as a "bargaining process" does not get at this goal-directed aspect of teaching. Fortunately, there is another definition which better captures this characteristic of instruction: To negotiate also means to "overcome obstacles skillfully" (as in, "carefully negotiating the winding road"). When two conditions are met, this aptly characterizes teaching for understanding: First, included under the rubric of obstacles are variables such as misconceptions or faulty reasoning that interfere with students' knowledge acquisition. Second, the process of overcoming these obstacles is viewed as a collaborative enterprise, shared by both students and teachers. The teacher's role, then, is akin to a guide's in helping students traverse new cognitive territory, pointing out, and working with them to overcome, potential obstacles to understanding. This might involve directly challenging students' naive conceptions. Work in science indicates that students must first be dissatisfied with their preconceptions before being receptive to alternative explanations (Posner, Strike, Hewson, & Gertzog, 1982).

The definition of teaching as the skillful, and collaborative, overcoming of obstacles contrasts with the traditional view. It represents a "cross-country" view of knowledge acquisition. Thus, according to Henry Pollack, most people think that acquiring expertise in a subject like mathematics involves carefully following a well-marked course: "Mathematics, as we teach it, is too often like walking on a path that is carefully laid out through the woods; it never comes up against any cliffs or thickets; it is all nice and easy" (cited in Lampert, in press, a). Pollack prefers an alternative view. According to this view, it is common and desirable for students to encounter obstacles as they attempt to negotiate the subject matter terrain.

Not surprisingly, teachers who play the role of guide by pointing out obstacles to students, probing the limits of their understanding with difficult cases or "entrapments" (i.e., questions designed to snare students into agreeing with certain erroneous ideas), frequently are viewed

by students as hinderers and not helpers in the learning process. Teachers will feel comfortable with this role only if they view uncertainty or conflict as an important, growth-producing commodity. There is evidence to show that teachers who embrace such a view are much better at fostering a strategic, mastery-oriented approach to learning (Dweck & Bempechat, 1983).

As indicated above, the teacher, as a guide, is expected to do more than point out potential obstacles to understanding. He or she is also expected to work collaboratively with students to help them overcome these obstacles. In playing this second role, the importance of having a cognitive map of the sort discussed in the previous section becomes immediately apparent. The teacher can be an effective guide only if he or she has a good sense of direction; not having a sure grasp of the cognitive territory one is to traverse puts teachers in the position of the "blind leading the blind." Teachers need to know where their instruction is heading; not, as Lampert (1988) puts it, "in the linear sense of one topic following another, but in the global sense of a network of big ideas and the relationships among those ideas, and facts, and procedures" (p. 163).

Having this sort of in-depth knowledge is necessary but not sufficient in equipping teachers to teach for understanding. One of the most important negotiatory skills for teachers appears to be that of structuring classroom discourse to promote knowledge organization and awareness in students. Not surprisingly, the ability to capitalize on this skill appears to relate to the depth of one's subject matter knowledge. Carlsen's (in press) study is one of several that supports this contention. He compared four high school science teachers teaching topics that they were either more or less knowledgeable about. When teaching unfamiliar topics, Carlsen's instructors tended to discourage student participation in discourse, both by talking more and by exercising tighter control. For example, they barely acknowledged correct student responses, and seldom allowed students to change the topic. These same techers, however, when teaching familiar topics, actively encouraged student input, frequently departing from the lesson plan to pursue individual questions and concerns.

Despite its important role in teaching for understanding, not much is known about how to structure discourse in the classroom. As Corno (1988) and Noddings (1985) point out, the cooperative learning techniques developed thus far, which stress the importance of group incentives and grades, seem most appropriate for lower level, achievement test outcomes; because they stress performance, these techniques may in fact reduce the likelihood that reflective dialogue will occur (Corno, 1988).

According to Roby (1988), it is the dialectical aspect of discourse that promotes student understanding. This aspect concentrates on articulating and contrasting student and teacher opinions. Unfortunately, Roby argues, much of what passes for discussion in classrooms is really "quasi-discussion." It lacks the reflective interaction of dialectical discourse. Quasi-discussions take two forms. One type, dominated by the teacher, follows a question–answer format; there is little opportunity for exchange of ideas. The other type is termed the "bull session." Here, students and their milieu dominate the topics of conversation; the rambling and uncoordinated discourse compares unfavorably with the purposiveness of dialectical discussion. Unlike the bull session, where there is a rhetorical winner and loser but no real attempt to resolve issues, those engaged in dialectic discussion seek common understandings:

> Opposing views become alternatives to be explored rather than competitors to be eliminated. Consensus on a large scale is not too much to hope for. The initial sense of rightness about one's own answers merges into sense of rightness about the process which scrutinizes all answers. (p. 173).

Dialectical discussions make use of a number of rhetorical devices, such as the "inviter" (e.g., "Would you tell us about it?") or the "prober" (e.g., "How has your view shifted from the opinion you gave earlier?") (Roby, 1988). One of the most important is the "parallel," which highlights similarities and differences. One type of parallel, for example, has students personalize academic problems by putting themselves in someone else's situation (e.g., "What strategy would you have followed had you been General Washington?"); other types of paral-

lels are more explicit in getting students to compare and contrast different viewpoints. By getting students to carefully examine parallels between their own viewpoints and those of others, the teacher educates students to the importance of connectedness in learning (Barnes, 1975).

In keeping with the negotiation metaphor, teachers play a different role in dialectical discussion. They function, in part, like moderators of discussion, facilitating student-student interaction and utilizing reflective or sustaining feedback to enhance the quality of the discussion (Klinzing & Klinzing-Eurich, 1988). They also provide critical feedback to students regarding the substance of their contributions. All of the above, of course, presupposes that teachers value, and take seriously, the contributions made by students.

Valuing student contributions is the first requirement for successful group work, according to Barnes (1975). It may form the basis for all genuine communication between teacher and student. Uhlenbeck (1978) asserts that it is difficult to exaggerate the importance of the hearer assuming some level of rationality on the part of the speaker. "The hearer always takes the view that what the speaker is saying somehow makes sense" (p. 190). This does not mean, however, that teachers should accept uncritically everything that students say—particularly when they evidence flaws in their thinking, or serious misconceptions that represent obstacles to understanding. Such mistakes need to be dealt with in as objective a way to possible. One way to do this is to depersonalize the mistake: Get students to view errors as natural, even useful, concomitants of learning rather than as occasions for embarrassment or shame (Dweck & Bempechat, 1983).

The distinction made earlier between the two types of "negotiation" is relevant here. Prior to engaging in collaborative, problem-solving activity (one definition), it is considered helpful for participants to reach some consensus about the nature of the undertaking (the other definition). As a result of this negotiation process, individuals can develop an appreciation for each other's roles and responsibilities. They also can establish the norms of interaction that will govern how members of the group relate to one another. This process is particularly important in the classroom (Cobb et al., in press). Agreeing on norms that minimize risk may be a necessary, if not sufficient, condition for collaboratively coming to terms with important impediments to understanding.

The outcome of the first type of negotiation process (i.e., the ground rules for discourse) strongly influences subsequent attempts to engage in the second type. Lampert's work (1987) supports this contention. As Lampert suggests, it may be that students need to learn that it is legitimate to have a meaningful discussion about content before they can learn from the discussion. Cobb et al. (in press), in their case study of a constructivist mathematics teacher, comment on the "dual structure" of classroom discourse. They argue that students in the classroom they observed were able to talk about mathematics in ways that facilitated understanding because norms that make such talk possible had been carefully negotiated at the beginning of the year. In getting students to adhere to these norms, the teacher was very direct in her interventions.

For example, in one situation where a child had inadvertently been put on the spot in front of the class, the teacher commented, "It's all right. Boys and girls, even if your answer is not correct, I am most interested in having you think. That's the important part. We are not always going to get answers right, but we want to try" (Cobb et al., in press). In other words, the teacher-led "talk about talking about mathematics" established a context conducive to collaborative problem-solving on the part of the students.

Lampert (in press b) also stresses the importance of establishing certain ground rules for classroom discourse. In her own mathematics teaching at the elementary level, she very consciously models patterns of discourse that parallel those used by scholars in the discipline. In working on problems, students are expected to recount their own reasoning processes and to analyse those of others. Lampert is quite particular about the language students use when they engage in this sort of discourse. When making assertions, for example, students are encouraged to say "I think" rather than "It is" or "I know:" "Saying 'I think' rather than 'It is' protects the student from associating his or her sense of self with an assertion that is later revised because it has been proven wrong".

Analysis/Diagnosis

In addition to the attributes talked about above, teaching for understanding can also be characterized by its highly analytic or diagnostic nature. This is less true of more traditional approaches to teaching. Student assessment has always been considered an important, but not primary, component of instruction (Putnam & Leinhardt, 1984); given the kinds of constraints under which teachers operate (e.g., dealing with 20 and 30 students), they appear to do a credible job of evaluating learning outcomes. Research shows, for example, that most teachers can accurately predict how their students will perform on individual test items, especially those items that are cognitively less complex (Coladarci, 1986). In more constructivist approaches to teaching and learning, however, the assessment, or more precisely, the analysis or diagnosis of student learning occupies an absolutely key position. Many researchers argue that analysis of student learning should be the basis for instructional decision making; clearly, it is now viewed as a more integral part of the teaching process.

As will become obvious, the need to constantly analyse what students are learning places a special burden on the teacher. At present, most teachers do not pay much attention to students' ideas and explanations during instruction (Smith & Neale, in press). This may reflect a lack of knowledge on their part. As with the other aspects of teaching for understanding, depth of subject-matter knowledge appears to play an important role in the analysis of student learning. Thus, Hashweh (1985) found that when teachers were knowledgeable about the topics they were teaching, they were able to generate a better set of questions to assess student understanding. On the other hand, when they lacked this knowledge, they often failed to recognize student errors; some, in the simulated teaching exercise Hashweh employed, even went so far as to reinforce student misconceptions.

In addition to content knowledge, teacher beliefs about learning influence the degree to which they focus on student comments and behavior. Arguably, the most important *general* knowledge for teachers to possess is that relating to the learning process. Studies show that teachers who subscribe to more of a constructivist view of learning attach greater importance to student input as a source of information about thinking than do teachers who embrace more of a traditional, absorptionist perspective. Thus, Peterson, Fennema, Carpenter, and Loef (1989) compared constructivist and non-constructivist teachers in mathematics; the former attended more to what students did and said during problem solving and thus had a more sophisticated understanding of the strategies their children used to solve simple word problems.

The best way to enhance assessment capability may be to provide teachers with fairly detailed information about children's thinking in specific subject matter domains. Those who stress the importance of this specific type of knowledge emphasize the need to effect a match between the intellectual resources children bring to a particular learning task and the cognitive demands of the instructional task (Romberg & Carpenter, 1986). Teachers can make intelligent decisions in this regard, the argument goes, only when they fully appreciate the developmental course of children's thinking in the subject matter domain. Roth (1987) tried this approach with a sample of 13 junior high school science teachers, with mixed results. Teachers were provided with information about student misconceptions as well as specific strategies they could use in dealing with this type of knowledge. Only a third of the teachers were successful in implementing the new strategies — a finding that is consistent with conceptual change theory (Roth, 1987). Several of the teachers, according to Roth's detailed analysis, merely went through the motions of conceptual change teaching without really buying into the view of learning that underlies the approach. These teachers used conceptual change strategies in unintended ways: "Their use of the strategy is best described as an empty use, because it did not engage students in making sense and constructing meaning of the concepts" (Roth, 1987, p. 10). Trying to get teachers to simultaneously change their views about what it means to learn *and* to teach a specific subject may require too much accommodation.

Researchers at the University of Wisconsin (Carpenter, Fennema, Peterson, Chiang &

Loef, 1989) have approached the assessment issue from a perspective similar to Roth's, although they have focused most of their energy on student cognitions. Rather than provide teachers with a program of instruction, they first familiarized teachers with research on the development of children's thinking about addition and subtraction. One of the purposes was to dispel the notion that number facts and computational skills must be mastered before children can solve word problems. In a recently completed study, teachers were encouraged to use this newly acquired information about children's invented strategies to design their own programs of instruction. As expected, the month-long, summer treatment phase of the study strongly affected teachers' orientations toward assessment; follow-up observations revealed that the experimental teachers, compared to the controls, elicited more, and were more attentive to students' explanations of their problem-solving strategies. Not surprisingly, researchers also found that the experimental teachers were more accurate in predicting the strategies their students would use to solve problems and generate number facts.

A third approach to getting teachers to attend more to student cognitions during mathematics has been utilized by Cobb et al. (1988). These researchers deliberately chose not to discuss models of early number learning with teachers during the first part of the study, arguing that teachers would not fully appreciate the relevance of these models to classroom practice at that point. Instead, the focus of their one-week summer institute was on classroom practice; this continued to be a major focus during the weekly, small-group follow-up phase of the study. These meetings addressed teachers' pragmatic concerns, such as how to involve children in mathematical discussions. The goal in dealing with issues of this sort was to get teachers to focus less on management concerns and more on the innovative mathematical activities that they were trying to implement.

It was thought that these changes in instructional practice would create a context that would make relevant the additional information about students that the researchers wanted to supply: "It was when the teachers began to use the problem-centered activities and encountered problematic situations that they came to realize that they had an inadequate knowledge of children's mathematics activity and actively wanted to learn about it" (Cobb et al., 1988, p. 30). This approach to developing analytic skills in teachers is less direct than the Carpenter et al. (1988) approach. Analysis of student learning is a by-product of getting teachers to fundamentally rethink their orientation to teaching a particular subject.

When teachers adopt a different set of instructional goals, it is hoped, they will find themselves attending to different kinds of student behavior. There is some indication that the process does unfold in this way. Putnam (1987), for example, found that teacher assessments were very much linked to the goals they pursued. Thus, the teachers in his study who favored an algorithmic approach to teaching addition focused more on students' ability to recite and carry out steps of the algorithm; teachers whose goals were more conceptual tended to emphasize student understanding of procedures — as reflected, for example, in the ability to link procedures to manipulatives.

Newman (1988) and his colleagues also observed a relationship between teachers' instructional goals — in this case, for high school social studies — and the kinds of behavior they attended to on the part of students. Teachers who placed the highest priority on student thinking were the most articulate when it came to dicussing what it involved. When asked to distinguish their best thinkers from other students, for example, these teachers' comparisons were lengthier, more detailed, and more elaborate than those provided by teachers who emphasized more traditional goals in the content domain.

As this discussion indicates, there are a number of ways to develop analytic skills in teachers. Each of the above has its adherents. Each could complement the other. A program aimed at getting teachers to be more analytic during instruction could emphasize all three: exposing teachers to the basic tenets of constructivism; providing teachers with detailed information about children's thinking in various subject matter domains; and encouraging teachers to experiment with, and carefully observe the effects of, different kinds of novel activities and curricular materials. This might exert a cumulative effect that none of the ap-

proaches, taken individually, could match.

Regardless of how one fosters analytic or diagnostic skills in teaching, however, there is a growing consensus that such skills are an essential component of teaching for understanding. Being analytic goes hand in glove with each of the other two attributes of this type of teaching. As Lampert (in press-a) points out, conjectures about student thinking should be part of the lesson planning process. Knowing what sorts of concepts or understandings are likely to be troublesome for students is important data for teachers to have when setting content priorities. Because the focus in this approach to assessment is less on the production of correct responses and more on the process of reasoning that underlies the responses, student learning is best analysed in an interactive context. Thus, it is important that the norms of interaction in the classroom actively encourage the public sharing of thought.

Conclusions

The approach to teaching presented in this paper differs dramatically from the traditional, "transmission" approach. Implementation would require wholesale changes in schools, teacher education, textbooks, and testing, of the sort called for in the Carnegie (1986) and Holmes Group (1986) reports. In teacher education, for example, much greater emphasis would be placed on providing teachers with rich networks of knowledge in mathematics, science, and other subject-matter domains, organized around the key ideas in each domain that could provide the conceptual basis for understanding in that domain. As the Holmes Report suggests, this alone would necessitate a sharp revision in the undergraduate arts and sciences curriculum. Too often, students are exposed only to the trees within each disciplinary field: They seldom get a chance to view the forest as a whole.

More attention must also be devoted to providing teachers with content-specific pedagogical knowledge of the sort described by Shulman and his colleagues (Shulman, 1986, 1987; Shulman & Sykes, 1986). If teachers are to help students "negotiate" new terrain in mathematics and science, they must develop an appreciation of those strengths and weaknesses students bring to this endeavor, and must have a mastery of the skills necessary to deal with those strengths and weaknesses. Knowing how students are likely to construe key ideas in the subject matter domain, what sorts of representation might aid students in their attempts to understand difficult ideas, how one goes about establishing "norms of discourse" in mathematics and science that minimize risk while allowing students to come to terms with important impediments to understanding—all are important aspects of teacher knowledge when teaching for understanding is the goal.

If we are to teach for understanding, new and better curricula have to be developed in mathematics and science. Tests need to be altered so that much greater emphasis is placed on students' understanding of key ideas and the ability to draw on or access this information in situations where it is potentially relevant (Prawat, in press). Finally, and perhaps most importantly, schools must be changed. Without changes in schools, it is unlikely that fundamental change will occur at the classroom level. This view, at least, is a basic tenet of the Holmes Group (1986) report, a document authored by educators dedicated to the reform of teacher education. Changes in teacher education alone are unlikely to improve the quality of learning in the classroom. In fact, the authors of the Holmes report insist, efforts to improve teacher education are inextricably bound to efforts to change how teachers work in schools. Not surprisingly, little progress has been made on either front. The nature and organization of teachers' work has changed little in the last 150 years:

> Many teachers still instruct whole classes of students in all subjects, as there is little or no academic specialization until high school. They still teach classes all day long, with little or no time for preparation, analysis, or evaluation of their work. They still spend all of their professional time alone with students, leaving little or no time for work with other adult professionals to improve their knowledge and skills. (Holmes Group, 1986, p.7).

It is obvious that it will not be easy to alter teaching practice in mathematics and science from the traditional focus on facts and procedures to an emphasis on conceptual level understanding. The stakes are high, however, and the potential payoff seems well worth whatever risks are involved.

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